CALCULUS:

MAY/JUNE 2019

QUESTION 7

7.1 Given $f(x) = x^2 + 2$.

Determine $f'(x)$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)

7.2.2 $y = 4\sqrt[3]{x} + (3x^3)^2$ (4)

7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \ldots$ (3)

[14]

NSC JUNE 2021

QUESTION 8

8.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 3x^2$. (5)

8.2 Determine:

8.2.1 $f'(x)$ if $f(x) = x^2 - 3 + \frac{9}{x^2}$ (3)

8.2.2 $g'(x)$ if $g(x) = (\sqrt{x} + 3)(\sqrt{x} - 1)$ (4)

[12]

NSC NOV 2020

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 1$. (5)

7.2 Determine:

7.2.1 $\frac{d}{dx}\left(\sqrt[5]{x^2} + x^3\right)$ (3)

7.2.2 $f'(x)$ if $f(x) = \frac{4x^2 - 9}{4x + 6}$; $x \neq -\frac{3}{2}$ (4)

[12]

NOV 2021

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x) = 2x^2 - 3x$. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = 4x^5 - 6x^4 + 3x$ (3)

9.2.2 $D_x\left[-\frac{\sqrt[3]{x}}{2} + \left(\frac{1}{3x}\right)^2\right]$ (4)

[12]

May/June 2019

QUESTION 8

A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x-axis at (–3 ; 0); $\left(-\frac{3}{2}\,;\,0\right)$ and (1 ; 0).

8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)

8.2 Calculate the x-coordinates of the turning points of h. (3)

8.3 Determine the value(s) of x for which h will be decreasing. (2)

8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)

[12]

JUNE 2017

QUESTION 9

Given: $f(x) = x^3 - x^2 - x + 1$

9.1 Write down the coordinates of the y-intercept of f. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (5)

9.3 Calculate the coordinates of the turning points of f. (6)

9.4 Sketch the graph of f in your ANSWER BOOK. Clearly indicate all intercepts with the axes and the turning points. (3)

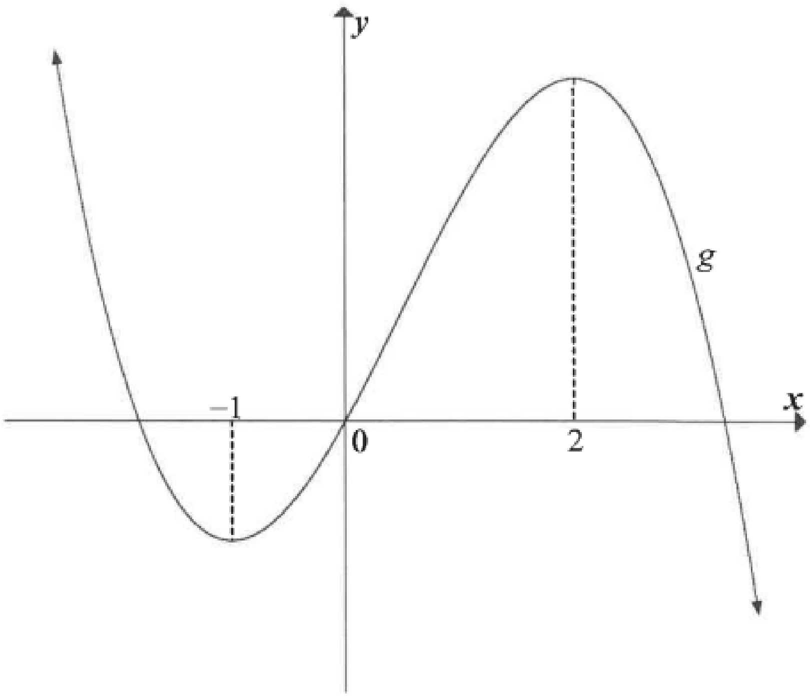
9.5 Write down the values of x for which $f'(x) < 0$. (2)

[17]

NOV 2020

QUESTION 8

The graph of $g(x)=ax^3+bx^2+cx$, a cubic function having a y-intercept of 0, is drawn below. The x-coordinates of the turning points of g are -1 and 2.



8.1 For which values of x will g increase? (2)

8.2 Write down the x-coordinate of the point of inflection of g. (2)

8.3 For which values of x will g be concave down? (2)

8.4 If $g'(x)=-6x^2+6x+12$, determine the equation of g. (4)

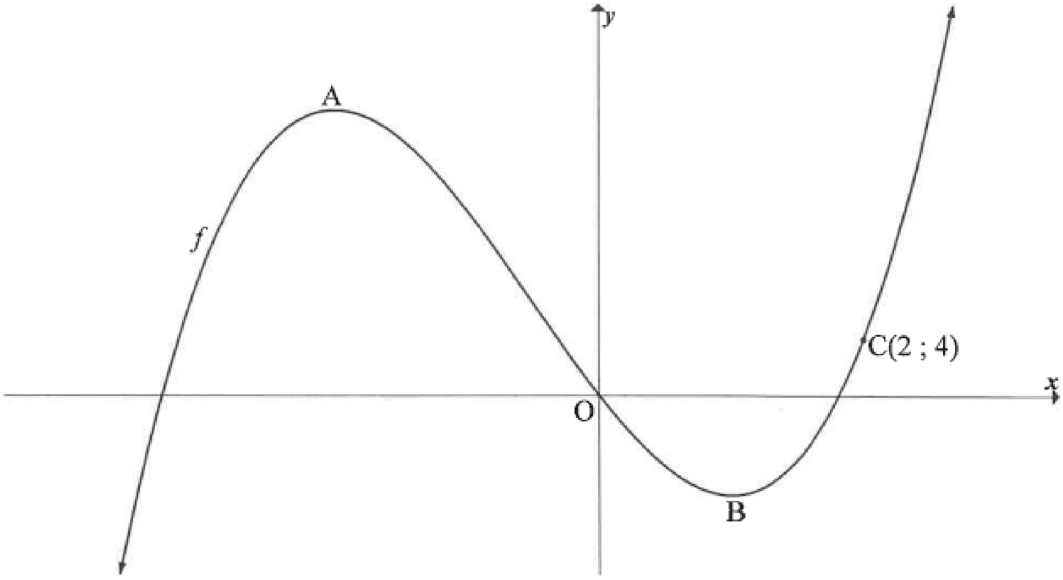
8.5 Determine the equation of the tangent to g that has the maximum gradient. Write your answer in the form $y=mx+c$. (5)

[15]

MAY/ June 2021

QUESTION 9

The graph of $f(x) = 2x^3 + 3x^2 - 12x$ is sketched below.
A and B are the turning points of f. C(2 ; 4) is a point on f.



9.1 Determine the coordinates of A and B. (5)

9.2 For which values of x will f be concave up? (3)

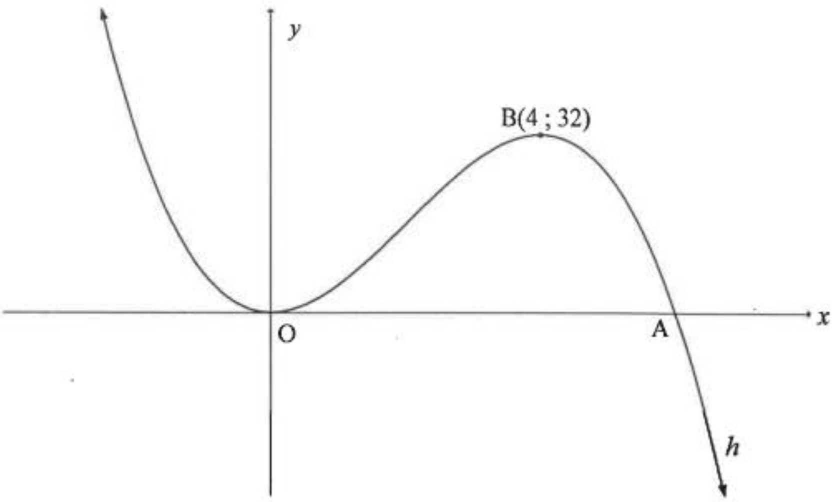
9.3 Determine the equation of the tangent to f at C(2 ; 4). (3)

[11]

NOV 2021

QUESTION 10

The graph of $h(x) = ax^3 + bx^2$ is drawn.
The graph has turning points at the origin, O(0 ; 0) and B(4 ; 32).
A is an x-intercept of h.

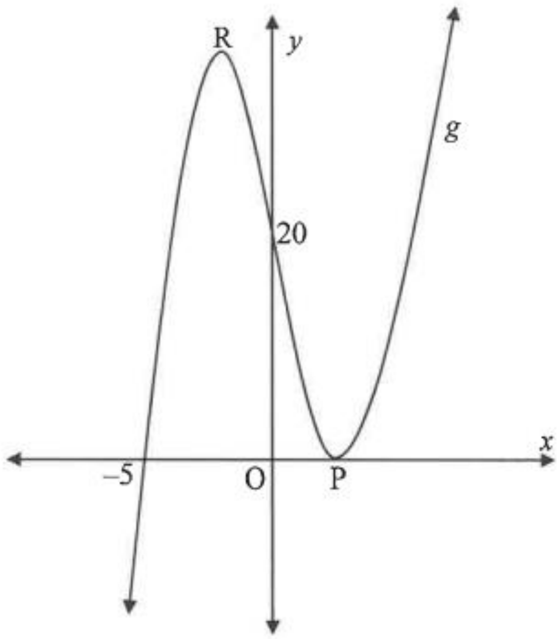


10.1	Show that $a = -1$ and $b = 6$.	(5)
10.2	Calculate the coordinates of A.	(3)
10.3	Write down the values of x for which h is:	
	10.3.1 Increasing	(2)
	10.3.2 Concave down	(2)
10.4	For which values of k will $-(x-1)^3 + 6(x-1)^2 - k = 0$ have one negative and two distinct positive roots?	(3)
		[15]

NOV 2018

QUESTION 9

9.1 The graph of $g(x) = x^3 + bx^2 + cx + d$ is sketched below.
The graph of g intersects the x-axis at $(-5 ; 0)$ and at P, and the y-axis at $(0 ; 20)$.
P and R are turning points of g.



9.1.1 Show that $b = 1$, $c = -16$ and $d = 20$. (4)

9.1.2 Calculate the coordinates of P and R. (5)

9.1.3 Is the graph concave up or concave down at $(0 ; 20)$? Show ALL your calculations. (3)

9.2 If g is a cubic function with:

- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

draw a sketch graph of g indicating ALL relevant points. (3)

[15]

JUNE 2021

QUESTION 10

10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

NOV 2019

QUESTION 9

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0 ; 0)$.

9.2.1 For which of the graphs will $(0 ; 0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)

[13]

NOV 2021

QUESTION 11

After travelling a distance of 20 km from home, a person suddenly remembers that he did not close a tap in his garden. He decides to turn around immediately and return home to close the tap.

The cost of the water, at the rate at which water is flowing out of the tap, is R1,60 per hour.

The cost of petrol is $\left(1{,}2+\frac{x}{4000}\right)$ rands per km, where x is the average speed in km/h.

Calculate the average speed at which the person must travel home to keep his cost as low as possible. **[7]**

NOV 2020

QUESTION 9

A closed rectangular box has to be constructed as follows:

- Dimensions: length (l), width (w) and height (h).
- The length (l) of the base has to be 3 times its width (w).
- The volume has to be 5 m^3.

The material for the top and the bottom parts costs R15 per square metre and the material for the sides costs R6 per square metre.

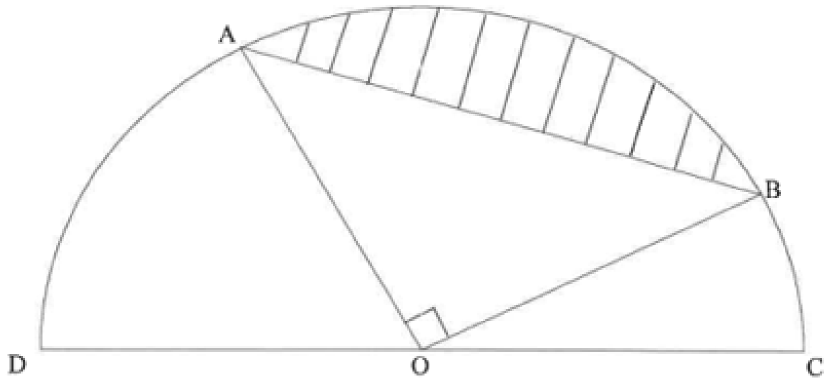
9.1 Show that the cost to construct the box can be calculated by: Cost $=90w^2+48wh$ (4)

9.2 Determine the width of the box such that the cost to build the box is a minimum. (6)

[10]

`JUNE 2021

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $(x - x^2)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$

(5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

[13]